

# Pure mathematics

June 2024 exam series

QUESTION PAPER

---

Provided for free student revision by Smart Edge International Ltd.  
Original material copyright OCR and its licensors. SmartEdge is not  
affiliated with or endorsed by OCR. For personal study only.  
Not for resale or redistribution.



Oxford Cambridge and RSA

**Tuesday 4 June 2024 – Afternoon**

**A Level Mathematics A**

**H240/01 Pure Mathematics**

**Time allowed: 2 hours**



**You must have:**

- the Printed Answer Booklet
- a scientific or graphical calculator

**QP**

### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ ms}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

### INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

### ADVICE

- Read each question carefully before you start your answer.

## Formulae

### A Level Mathematics A (H240)

#### Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

#### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

#### Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

#### Differentiation

| $f(x)$                   | $f'(x)$                          |
|--------------------------|----------------------------------|
| $\tan kx$                | $k \sec^2 kx$                    |
| $\sec x$                 | $\sec x \tan x$                  |
| $\cot x$                 | $-\operatorname{cosec}^2 x$      |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

#### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

#### Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

#### Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , mean of  $X$  is  $np$ , variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

|     |       |       |       |       |       |       |        |       |        |
|-----|-------|-------|-------|-------|-------|-------|--------|-------|--------|
| $p$ | 0.75  | 0.90  | 0.95  | 0.975 | 0.99  | 0.995 | 0.9975 | 0.999 | 0.9995 |
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807  | 3.090 | 3.291  |

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

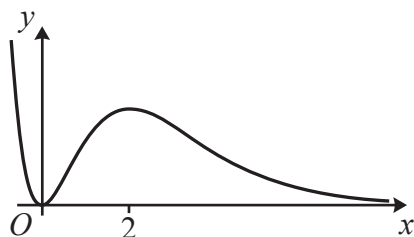
$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

1



The diagram shows part of the curve  $y = x^2 e^{-x}$ .

- (a) Use the trapezium rule with 4 intervals of equal width to find an estimate for  $\int_0^2 x^2 e^{-x} dx$ .  
Give your answer correct to 3 significant figures. [4]
- (b) Explain how the trapezium rule could be used to obtain a more accurate estimate for  $\int_0^2 x^2 e^{-x} dx$ . [1]
- (c) Explain why it is not clear from the diagram whether the value from part (a) is an under-estimate or an over-estimate for  $\int_0^2 x^2 e^{-x} dx$ . [2]

- 2 You are given that  $y$  is inversely proportional to  $x^6$  and  $z$  is directly proportional to the cube root of  $y$ .

- (a) (i) Find an equation for  $z$  in terms of  $x$  and  $k$ , where  $k$  is a constant of proportionality. [2]
- (ii) State which of the diagrams below could represent the graph of  $z$  against  $x$ . [1]

Fig. 1.1

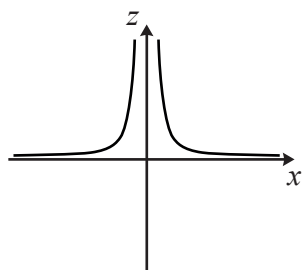


Fig. 1.2

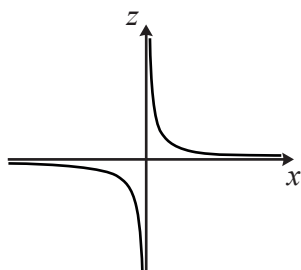


Fig. 1.3

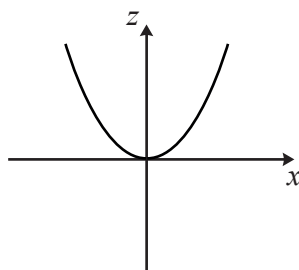
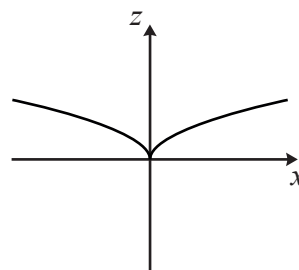


Fig. 1.4



- (b) Given that  $z = 3$  when  $x = 4$ , determine the values of  $x$  when  $z = 12$ . [3]

- 3 (a) Find a counterexample to disprove the statement that the product of two prime numbers is always odd. [1]

- (b) In each of the following cases write one of the symbols  $\Rightarrow$ ,  $\Leftrightarrow$ ,  $\Leftarrow$  in the box in the Printed Answer Booklet to make each statement correct.

(i)  $x^2 = 3x$    $x = 3$  [1]

(ii)  $x > 4$    $x^3 > 64$  [1]

(iii)  $x^\circ = 45^\circ$    $\tan x^\circ = 1$  [1]

- (c) Prove that the sum of the squares of **any** two odd numbers is always a multiple of 2 but never a multiple of 4. [4]

- 4 A sequence has terms  $u_1, u_2, u_3, \dots$  defined by  $u_1 = 2$  and  $u_{n+1} = 1 - \frac{1}{u_n}$  for  $n \geq 1$ .

- (a) Find the values of  $u_2, u_3$  and  $u_4$ . [2]

- (b) Describe the behaviour of the sequence. [1]

- (c) Given that  $\sum_{n=1}^k u_n = 73$ , determine the value of  $k$ . [3]

- 5 The line  $x + 13y = 108$  is the normal to the curve  $y = ax^2 + b\sqrt{x}$  at the point (4, 8).

Determine the values of the constants  $a$  and  $b$ . [8]

- 6 In this question you must show detailed reasoning.

The cubic polynomial  $f(x)$  is defined by  $f(x) = 4x^3 - 25x^2 - 58x + 16$ .

- (a) Show that  $x = \frac{1}{4}$  is a root of the equation  $f(x) = 0$ . [1]

- (b) Hence express  $f(x)$  as the product of a linear factor and a quadratic factor, with all terms in the factors having integer coefficients. [3]

- (c) Solve the equation  $4e^{3y} - 25e^{2y} - 58e^y + 16 = 0$ , giving each root in the form  $y = k \ln 2$  where  $k$  is a constant. [4]

7 The point  $A$  has coordinates  $(1, 7)$ , and the point  $B$  has coordinates  $(h, 10)$ .

- (a) You are given that the gradient of the line  $AB$  is 2.

Find the value of  $h$ . [2]

- (b) You are given that  $B$  is the midpoint of  $AC$ .

Find the coordinates of the point  $C$ . [2]

- (c) You are given that the straight line through the points  $A$ ,  $B$  and  $C$  has two distinct points of intersection with the curve  $y = x^2 - 4x + k$ .

Determine the set of possible values of  $k$ . [6]

- 8 (a) State the set of values for which  $|x| > x$ . [1]

- (b) You are given that  $n$  is an integer such that  $|n| \leq 9$ .

(i) Find the maximum value of  $|2n - 1|$ . [1]

(ii) Find the minimum value of  $|2n - 1|$ . [1]

- (c) (i) Solve the equation  $\left|\frac{1}{2}x - 1\right| = |2x - 3|$ . [3]

(ii) Explain why the equation  $\left|\frac{1}{2}x - 1\right| = 2x - 3$  has only one solution, and state the value of this solution. [1]

- 9 The depth of the water,  $d$  metres, in a tidal river during a given day is modelled by the equation

$$d = 1.9 + 1.1 \cos(30t - 60)^\circ$$

where  $t$  is the number of hours after midnight.

(A tidal river is one whose level is influenced by tides.)

- (a) (i) Find the minimum depth of water given by this model. [1]

- (ii) Find the value of  $t$  when the minimum depth first occurs. [2]

- (b) A boat can only enter the river when the depth of water is at least 1 metre.

Determine the two periods of time during the day between which this boat will **not** be able to enter the river. Give your answers correct to the **nearest minute**. [5]

In reality the depth of the river decreases as this boat travels along the river. An improved model uses the equation

$$d = e^{-cp}(1.9 + 1.1 \cos(30t - 60)^\circ)$$

where  $c$  is a positive constant and  $p$  is the distance, in kilometres, travelled along the river after entering it.

- (c) Explain how this new equation could give an improved model. [1]

10 In this question you must show detailed reasoning.

The first three terms of a convergent geometric progression are  $2x + 3$ ,  $x + 9$  and  $2x - 6$  respectively.

Determine the sum to infinity of this geometric progression. [8]

**Turn over for questions 11 and 12**

11 A curve has equation  $y = 5 \ln(1 - \cos 2x)$ , where  $x$  is in radians.

(a) State the values of  $x$  for which  $5 \ln(1 - \cos 2x)$  is not defined. [2]

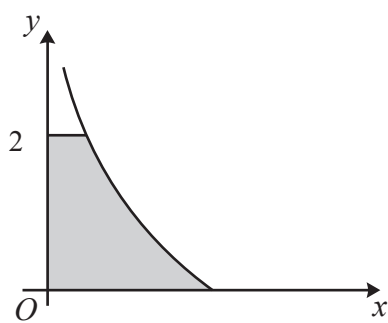
(b)  $P$  is the stationary point on the curve that has the smallest positive  $x$ -coordinate.

Determine the exact coordinates of  $P$ . [4]

(c) (i) Show that  $\frac{d^2y}{dx^2} + 20e^{-\frac{1}{5}y} = 0$ . [5]

(ii) State what can be deduced about all of the stationary points on this curve, giving a reason for your answer. [1]

12 In this question you must show detailed reasoning.



The diagram shows the curve with parametric equations  $x = \frac{2}{(2t+1)^4}$ ,  $y = 2t^2 + 3t$  for  $t \geq 0$ .

The shaded region is enclosed by the curve, the  $x$ -axis, the  $y$ -axis and the line  $y = 2$ .

(a) Show that the area of the shaded region is given by  $\int_a^b \frac{8t+6}{(2t+1)^4} dt$ , where  $a$  and  $b$  are constants to be determined. [5]

(b) Determine the exact area of the shaded region. [6]

**END OF QUESTION PAPER**

**OCR**

Oxford Cambridge and RSA

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.